

UNIT-IV
COMPLEX DIFFERENTIATION

S.No	Questions	BT	CO	PO
Part – A (Short Answer Questions)				
1	Find all values of k such that $f(z)=e^x(\cos y + i \sin y)$ is analytic.	L1	CO4	Po1
2	Show that z^2 is analytic for all z.	L1	CO4	Po1
3	Show that the function $f(z)=\sin x \cosh y + i \cos x \sinh y$ is continuous as well as analytic everywhere.	L1	CO4	Po1
4	Define Cauchy-Reimann equations in polar form.	L1	CO4	Po1
5	Define harmonic function.	L1	CO4	Po1
6	Define analytic function.	L1	CO4	Po1
7	Separate the real and imaginary parts of $\cot z$.	L1	CO4	Po1
8	Find the real and imaginary parts of e^{z^2}	L1	CO4	Po1
9	Prove that $\sin \bar{z} = \overline{\sin z}$	L1	CO4	Po1
10	Find all the roots of the equation $\sin z = 2$.	L2	CO4	Po1
Part – B (Long Answer Questions)				
11	a) State Cauchy-Riemann equation and if $f(z)$ is a regular function of z, prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] f(z) ^2 = 4 f'(z) ^2$	L2	CO4	Po1
	b) If $\tan(\log(x+iy)) = a+ib$, where $a^2+b^2 \neq 1$ show that $\frac{2a}{1-a^2-b^2} = \tan(\log(x^2+y^2))$.	L3	CO4	Po1
12	a) Define analytic function and show that both the real and imaginary parts of an analytic function satisfied Laplace's equation (are harmonic).	L2	CO4	Po1
	b) Define polar form of Cauchy-Riemann equation and prove that z^n is analytic and hence find its derivative.	L2	CO4	Po1
13	Prove that the function $f(z)$ defined by $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous and the Cauchy-Riemann equation are satisfied at the origin, yet $f'(0)$ does not exist.	L2	CO4	Po1
14	a) If $\tan(x+iy) = A + iB$, then prove that $A^2+B^2-2B\coth 2y+1=0$	L3	CO4	Po1
	b) Find k such that $f(x,y)=x^3+3kxy^2$ may be harmonic and find its conjugate.	L3	CO4	Po1
15	a) separate the real and imaginary parts of (i) $\cot z$ (ii) $\tanh z$.	L2	CO4	Po1
	b) Show that $u(x,y) = e^{2x}(x \cos 2y - y \sin 2y)$ is harmonic and find its harmonic conjugate.	L3	CO4	Po2
16	a) Prove that $u = e^{-x}[(x^2 - y^2)\cos y + 2xysin y]$ is harmonic and find the analytic function whose real part is u	L3	CO4	Po1
	b) Find the analytic function whose imaginary part is $\frac{2\sin x \sin y}{\cos 2x + \cosh 2y}$	L3	CO4	Po1